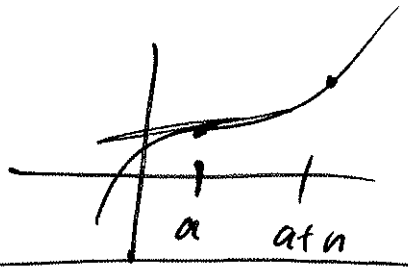
THE INSTANTANEOUS RATE OF CHANGE OF $f(x)$ AT a IS ITS DERIVATIVE

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



FOR EXAMPLE IF $h(t)$ IS A POSITION FUNCTION $h'(t)$ IS A VELOCITY FUNC

FOR QUANTITIES THAT ARE CHANGING OVER TIME, THE RATES AT WHICH THESE QUANTITIES CHANGE ARE GIVEN BY DERIVATIVES.

IF TWO RELATED QUANTITIES ARE CHANGING OVER TIME, THE RATES AT WHICH THESE QUANTITIES CHANGE ARE RELATED.

Related rates

FOR EXAMPLE, IF A BALLOON IS BEING FILLED WITH AIR, BOTH THE RADIUS OF THE BALLOON AND THE VOLUME OF THE BALLOON ARE INCREASING.

WE WANT TO DETERMINE THE RELATIONSHIP BETWEEN THE RATES OF CHANGE OF THESE QUANTITIES.

WE CALL OUR STUDY **RELATED RATES.**

10/8/19 CALCULUS ONE - RELATED RATES

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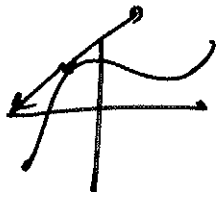
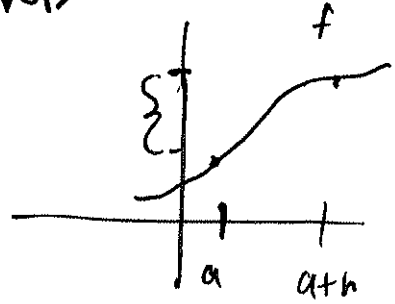
DERIVATIVES CALCULATE THE SLOPE OF THE TANGENT LINE TO THE GRAPH OF A FUNCTION AT A GIVEN POINT.



IN OTHER WORDS, THE DERIVATIVE IS THE <u>INSTANTANEOUS RATE OF CHANGE</u> OF THE FUNCTION.

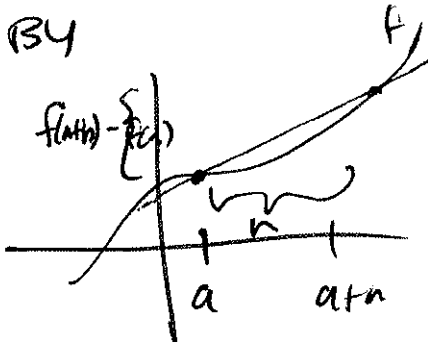
IF $f(x)$ IS A FUNCTION DEFINED ON AN INTERVAL $[a, a+h]$, THEN THE <u>AMOUNT OF CHANGE</u> OF $f(x)$ OVER THE INTERVAL IS THE CHANGE IN THE y VALUES OF THE FUNCTION OVER THAT INTERVAL AND IS GIVEN BY

$$f(a+h) - f(a)$$



THE <u>AVERAGE RATE OF CHANGE</u> OF THE FUNCTION f OVER THE SAME INTERVAL IS THE RATIO OF THE AMOUNT OF CHANGE OVER THAT INTERVAL TO THE CORRESPONDING CHANGE IN x VALUES AND IS GIVEN BY

$$\frac{f(a+h) - f(a)}{h}$$



10/8/19 C1-RR

WE'LL:

- EXPRESS CHANGING QUANTITIES IN TERMS OF DERIVATIVES
- FIND RELATIONSHIPS AMONG DERIVATIVES IN A GIVEN PROBLEM
- USE THE CHAIN RULE TO FIND THE RATE OF CHANGE OF ONE QUANTITY THAT DEPENDS ON THE RATE OF CHANGE OF OTHER QUANTITIES.

SETTING UP RELATED-RATES PROBLEMS

IN MANY REAL-WORLD APPLICATIONS, RELATED QUANTITIES ARE CHANGING WITH RESPECT TO TIME.

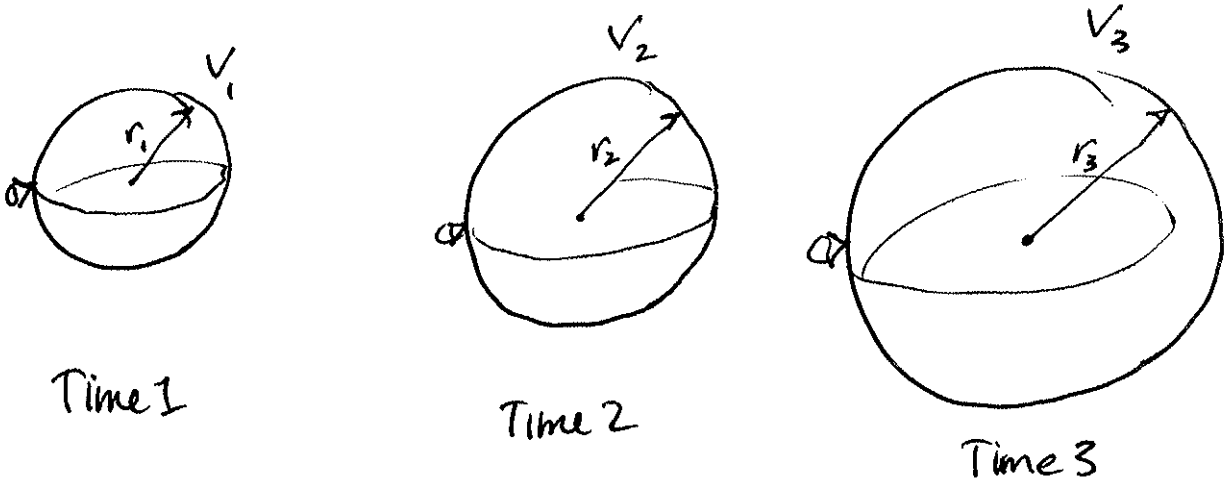
FOR EXAMPLE, IN THE BALLOON EXAMPLE, WE CAN SAY THAT THE RATE OF CHANGE IN THE VOLUME V, IS RELATED TO THE RATE OF CHANGE IN THE RADIUS, r.

IN THIS CASE, WE SAY THAT $\frac{dV}{dt}$ AND $\frac{dr}{dt}$ ARE **RELATED RATES** BECAUSE V IS RELATED TO r.

EX. INFLATING A BALLOON

A SPHERICAL BALLOON IS BEING FILLED WITH AIR AT THE CONSTANT RATE OF $2 cm^3/sec$.

HOW FAST IS THE RADIUS INCREASING WHEN THE RADIUS IS 3cm?



Time 1 Time 2 Time 3

THE VOLUME OF A SPHERE OF RADIUS r CENTIMETERS IS

$$V = \frac{4}{3}\pi r^3 \quad cm^3$$

SINCE THE BALLOON IS BEING FILLED WITH AIR, BOTH THE VOLUME AND THE RADIUS ARE FUNCTIONS OF TIME.

THEREFORE, t SECONDS AFTER BEGINNING TO FILL THE BALLOON WITH AIR, THE VOLUME OF AIR IN THE BALLOON IS

$$V(t) = \frac{4}{3}\pi [r(t)]^3$$

DIFFERENTIATING BOTH SIDES OF THE EQUATION WITH RESPECT TO TIME & APPLYING THE CHAIN RULE, WE SEE THAT ITS RATE OF CHANGE IN THE VOLUME IS RELATED TO THE RATE OF CHANGE IN THE RADIUS BY THE EQUATION

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3[r(t)]^2 \cdot \frac{dr}{dt}$$

THE BALLOON IS BEING FILLED WITH AIR AT THE CONSTANT RATE OF $2\,cm^3/s$, SO $\frac{dV}{dt} = 2\,cm^3/s$

THEREFORE

$$2 \text{ cm}^2/\text{s} = 4\pi [r(t)]^2 \cdot \frac{dr}{dt}$$

WHICH IMPLIES

$$\frac{dr}{dt} = \frac{1}{2\pi [r(t)]^2} \text{ cm/s}$$

WHEN THE RADIUS $r = 3\text{cm}$

$$\frac{dr}{dt} = \frac{1}{2\pi [3]^2} \text{ cm/s}$$

HOW FAST IS THE radius changing when $r = 6\text{cm}$?

10/8/19 C1 - R.R.

PROBLEM SOLVING STRATEGY : RELATED RATES

① ASSIGN SYMBOLS TO ALL VARIABLES INVOLVED IN THE PROBLEM. DRAW A FIGURE IF APPLICABLE.

② STATE, IN TERMS OF VARIABLES, ~~THE INFORMATION~~ THAT IS GIVEN & THE RATE TO BE DETERMINED

③ FIND AN EQUATION RELATING THE VARIABLES INTRODUCED IN STEP 1.

④ USING THE CHAIN RULE, DIFFERENTIATE BOTH SIDES OF THE EQUATION FOUND IN STEP 3 WITH RESPECT TO THE INDEPENDENT VARIABLE.

* THIS NEW EQUATION WILL RELATE THE DERIVATIVES *

⑤ SUBSTITUTE ALL KNOWN VALUES INTO THE EQUATION FROM STEP 4, THEN SOLVE FOR THE UNKNOWN RATE OF CHANGE.

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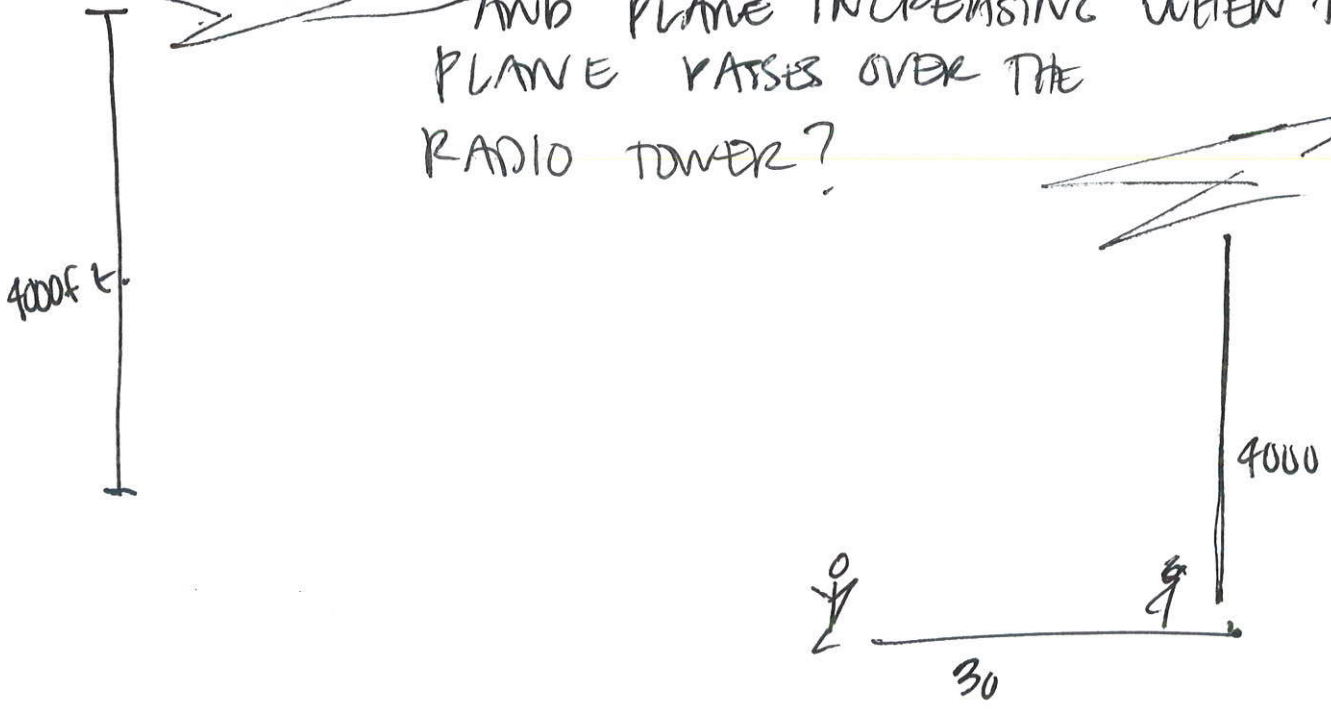
EX. AN AIRPLANE FLYING AT A CONSTANT ELEVATION

AN AIR PLANE IS FLYING OVERHEAD AT A CONSTANT ELEVATION OF 4000 ft. A MAN IS VIEWING THE PLANE FROM A POSITION 3000 ft FROM THE BASE OF A RADIO TOWER.

THE PLANE IS FLYING HORIZONTALLY AWAY FROM THE MAN.

IF THE PLANE IS FLYING AT THE RATE OF 600 ft/sec AT WHAT RATE IS THE DISTANCE BETWEEN THE MAN AND PLANE INCREASING WHEN THE PLANE PASSES OVER THE RADIO TOWER?



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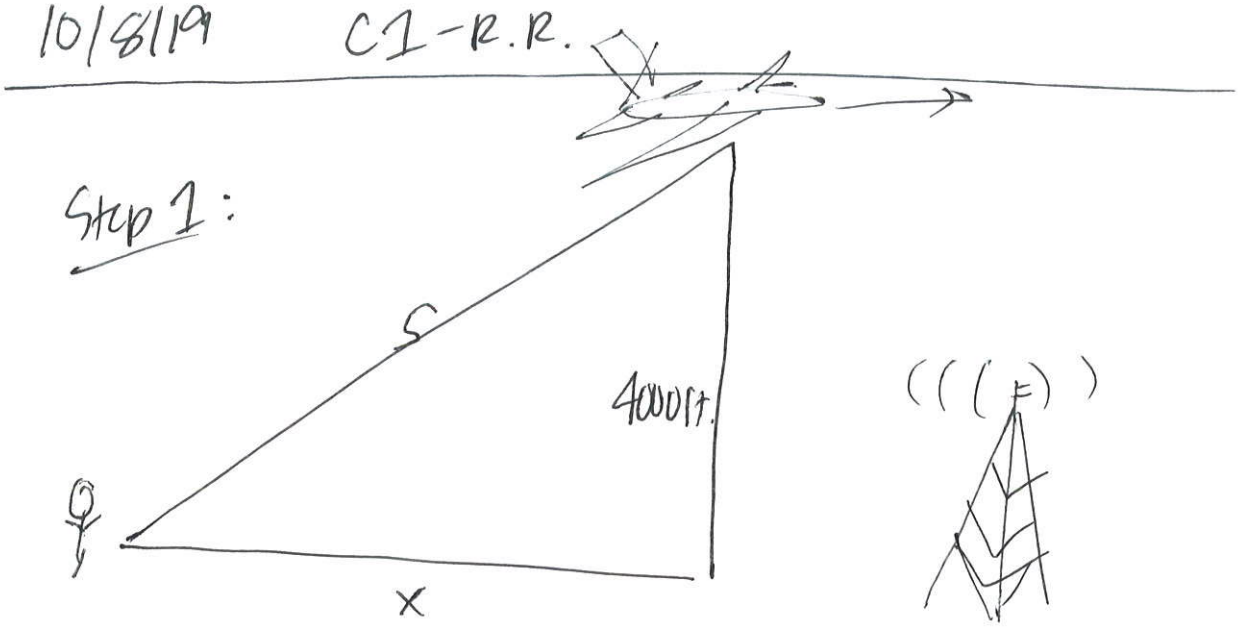

Step 2:

$\frac{dx}{dt}$ = Speed of plane = 600 ft/s

want to find $\frac{ds}{dt}$ when $x = 3000$

Step 3 equation involving x and s:

PYTHAGOREAN THEOREM: $x^2 + 4000^2 = s^2$

Step 4

$$2x \cdot \frac{dx}{dt} + 0 = 2s \cdot \frac{ds}{dt}$$

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Step 5

$$2x\frac{dx}{dt} = 2s\frac{ds}{dt}$$

Given $x = 3000$ ft

$\frac{dx}{dt} = 600$ ft/s

FIND $s = \sqrt{(3000)^2 + (1000)^2} = 5000$ ft

$$2(3000)(600) = 2(5000)\frac{ds}{dt}$$

$$\frac{ds}{dt} = 360 \text{ ft/s}$$

What is the speed of the plane ~~when the~~ if ~~person and the~~ the distance between the person and the plane is increasing at the rate of 300 ft/s?