

Review for Multi II Spring 2019

1. Compute the divergence and curl of

$$\mathbf{F}(x, y, z) = \langle 3x + 2z^2, \frac{x^3 y^2}{z}, -(z - 7x) \rangle$$

2. Is the following vector field conservative?

$$\mathbf{F} = \langle 4y^2 + \frac{3x^2 y}{z^2}, 8xy + \frac{x^3}{z^2}, 11 - \frac{2x^3 y}{z^3} \rangle$$

For problems 3-5, write down a vector equation which describes the surface (or set of parametric equations).

3. The plane  $7x + 3y + 4z = 15$ .

4. The cylinder  $x^2 + y^2 = 5$  for  $-1 \leq z \leq 6$ .

5. The portion of the sphere of radius 6 with  $x \geq 0$ .

6. Determine the surface area of the plane  $2x + 3y + 6z = 9$  that is inside the cylinder  $x^2 + y^2 = 7$ .

7. Evaluate  $\int \int_S z + 3y - x^2 \, dS$  where  $S$  is the part of the  $z = 2 - 3y + x^2$  that lies over the triangle in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(2, 0)$  and  $(2, -4)$ .

8. Calculate the line integral

$$\int x^2 y \, dx + (y - 3) \, dy$$

where  $C$  is a rectangle with vertices  $(1, 1)$ ,  $(4, 1)$ ,  $(4, 5)$  and  $(1, 5)$  oriented counterclockwise.

9. Calculate the line integral  $\int_C \mathbf{F} \cdot d\mathbf{r}$ , where

$$\mathbf{F} = \langle xy, x^2 + y^2 + z^2, yz \rangle$$

and  $C$  is the boundary of the parallelogram with vertices  $(0, 0, 1)$ ,  $(0, 1, 0)$ ,  $(2, 0, -1)$  and  $(2, 1, -2)$ .

10. Calculate the surface integral  $\int \int_S \mathbf{F} \cdot d\mathbf{S}$ , where  $S$  is the cylinder  $x^2 + y^2 = 1$ , with  $0 \leq z \leq 2$ , including the circular top and bottom, and  $\mathbf{F} = \langle \frac{x^3}{3} + yz, \frac{y^3}{3} - \sin(xz), z - x - y \rangle$ .